

Applications of Vectors in Geometry

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Summary of last time

● Last time:

- vector (cross) product
- vector product formula in components
- applications in mechanics (moments, rotation)
- scalar triple product
- vector triple product

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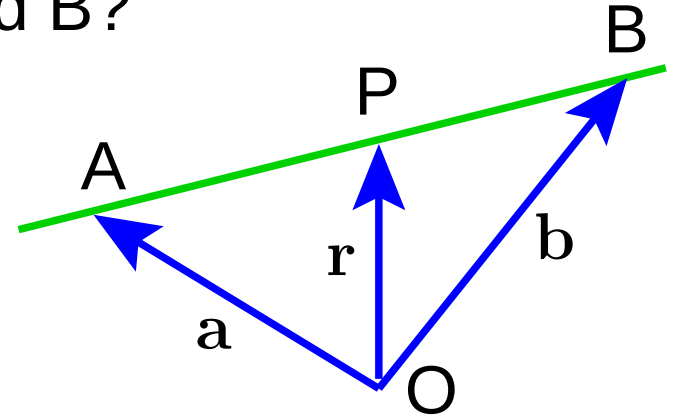
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- vector product formula in components
- applications in mechanics (moments, rotation)
- scalar triple product
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● This time: geometry

- vector equations of line, plane and sphere
- how to differentiate vectors!

Vector equation of a line

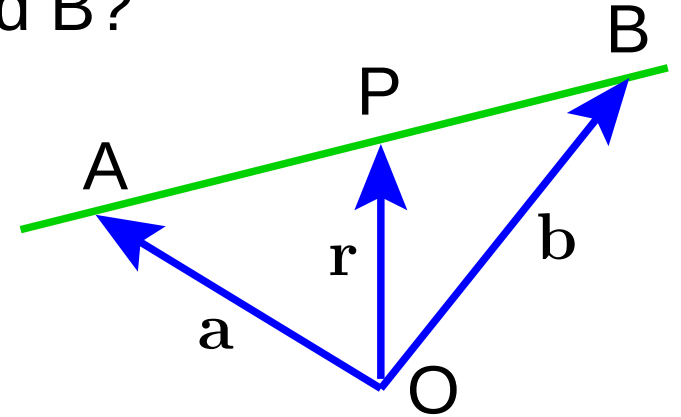
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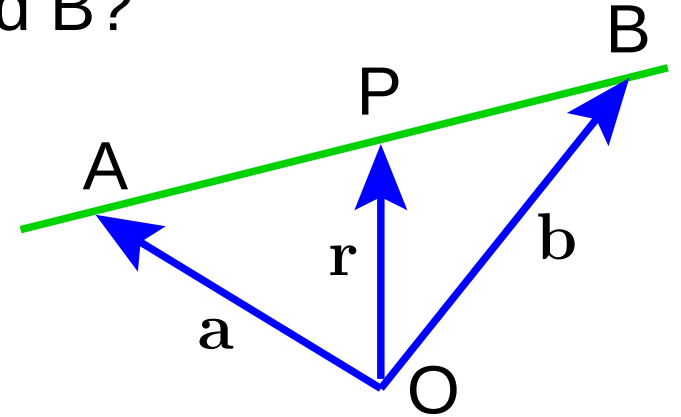
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so $\vec{AP} = t \vec{AB}$ (scalar t)
so $\mathbf{r} - \mathbf{a} = t(\mathbf{b} - \mathbf{a})$.



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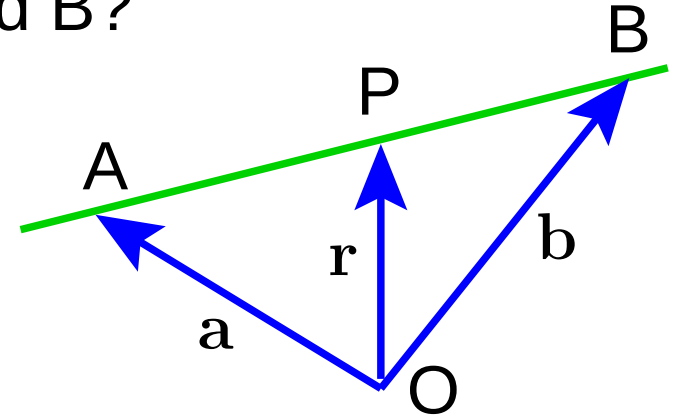
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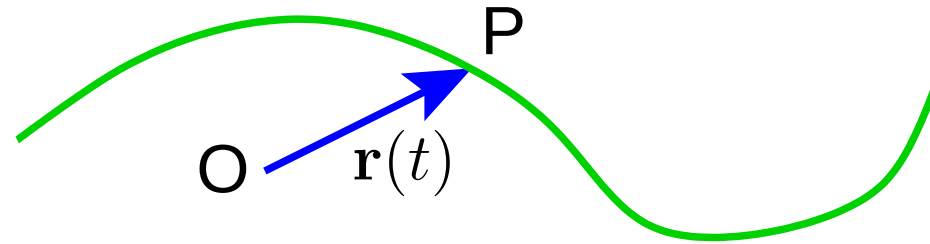
$$\mathbf{r} = \mathbf{a} + t(\mathbf{b} - \mathbf{a})$$

- As t varies, P moves along line
 - E.g., when $t = 0$, $P=A$
 - Or when $t = 1$, $P=B$
 - t varies from $-\infty$ to $+\infty$

Exercise

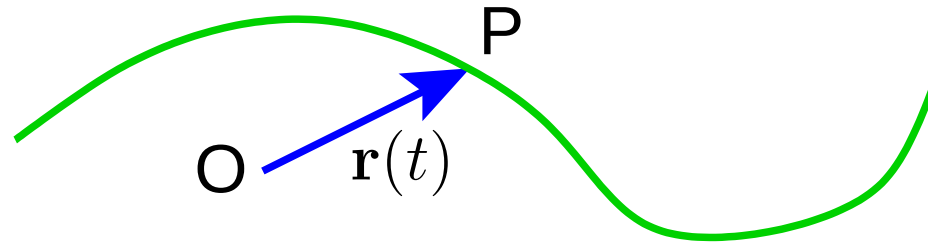
- *James* p239
- $\mathbf{a} = (1, 4, 6)$ and $\mathbf{b} = (3, 5, 7)$ lie on the same straight line.
- Find
 - the equation of the line
 - where it intersects the (x, z) plane

Differentiation of vectors



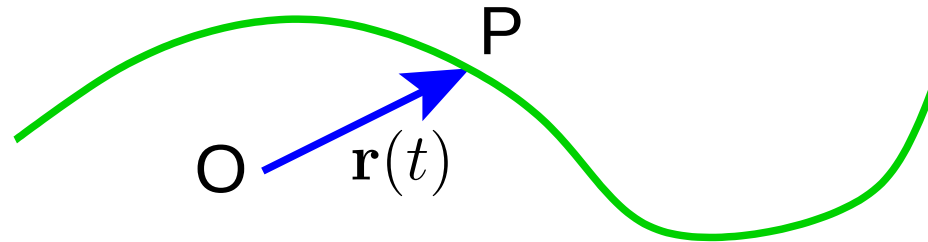
- Curvy line $\mathbf{r}(t)$:
natural interpretation as position of particle at time t
- Q: How to find velocity $\frac{d\mathbf{r}}{dt}$?

Differentiation of vectors



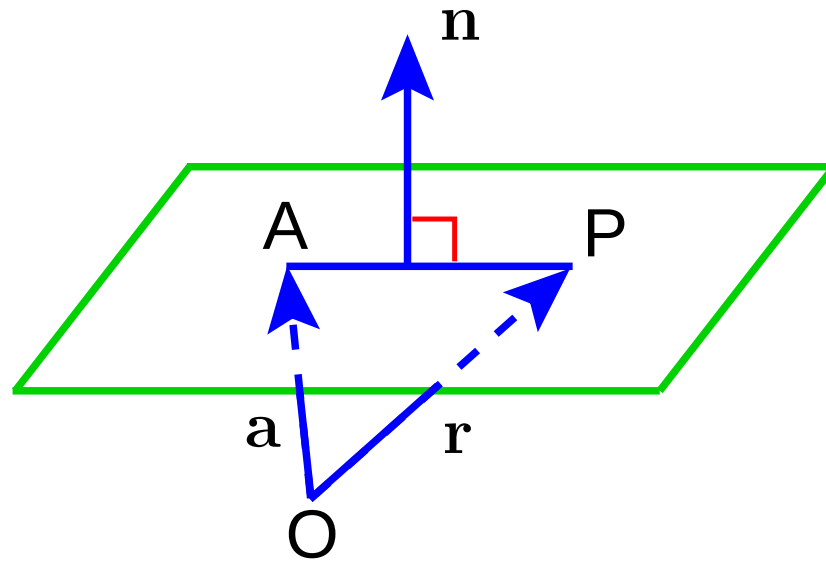
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A: Just differentiate component by component.
- Example: Take $\mathbf{r} = (\log_e t, \sin t, t^3 + 3t^2)$. Find $\frac{d\mathbf{r}}{dt}$.

Differentiation of vectors



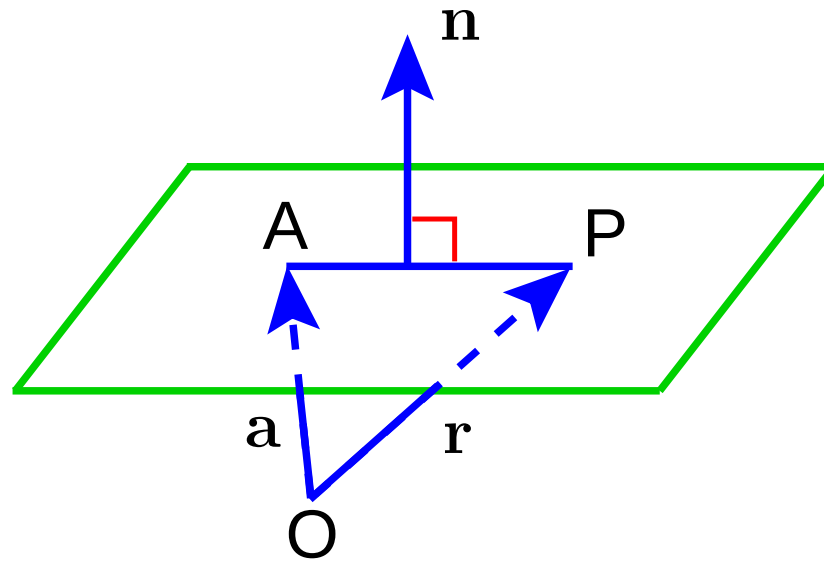
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- More complicated vector differentiation next year

Vector equation of a plane



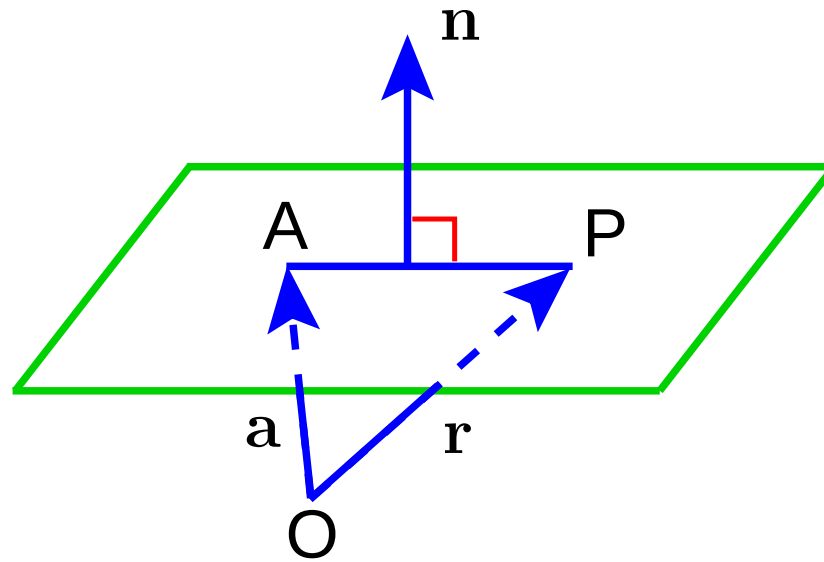
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Vector equation of a plane



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- Vector equation of plane:

$$\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$$

or (alternative)

$$\mathbf{r} \cdot \mathbf{n} = \rho \quad (\rho \text{ scalar constant})$$

Exercise

- *James* p244
- Find the point where the plane

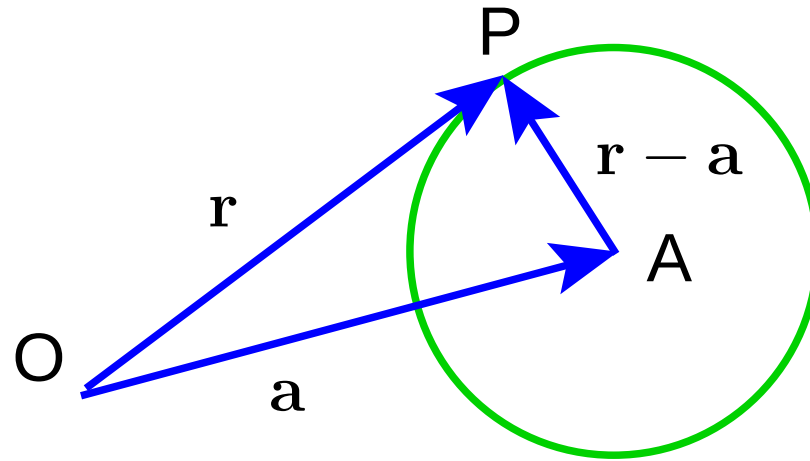
$$\mathbf{r} \cdot (1, 2, 2) = 3$$

meets the line

$$\mathbf{r} = (2, 1, 1) + \lambda(0, 1, 2)$$

Vector equation of a sphere

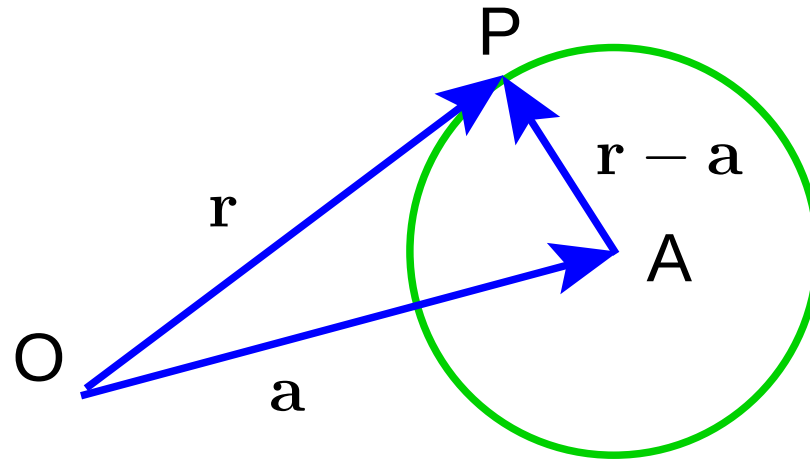
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Vector equation of a sphere

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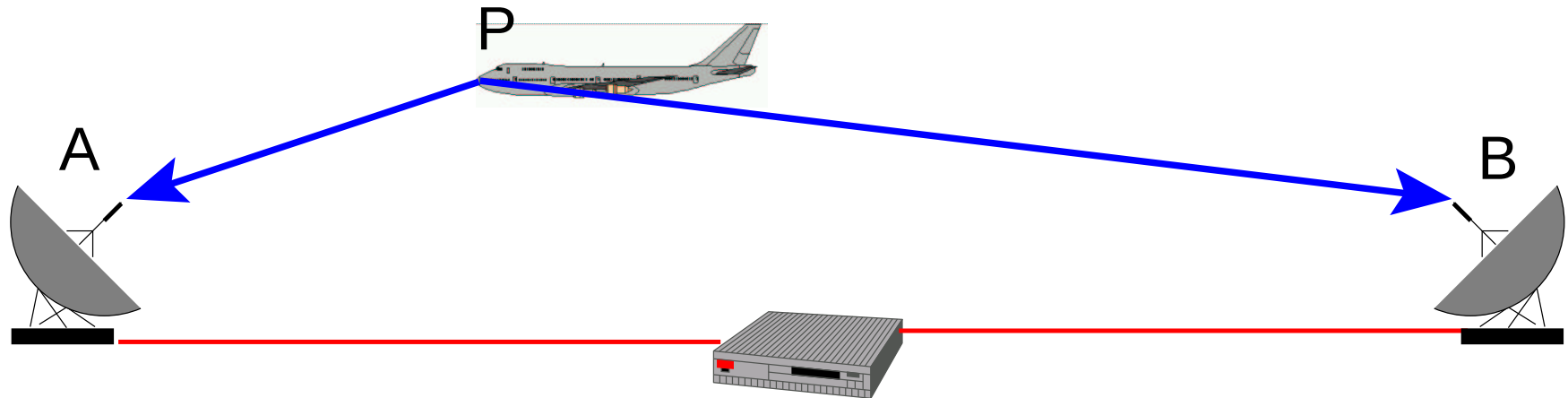


- P always same distance (radius ρ_s) from A
- NB: $\vec{AP} = \mathbf{r} - \mathbf{a}$, so $|\vec{AP}| = |\mathbf{r} - \mathbf{a}|$
- Vector equation of circle (or sphere in 3D):

$$|\mathbf{r} - \mathbf{a}| = \rho_s$$

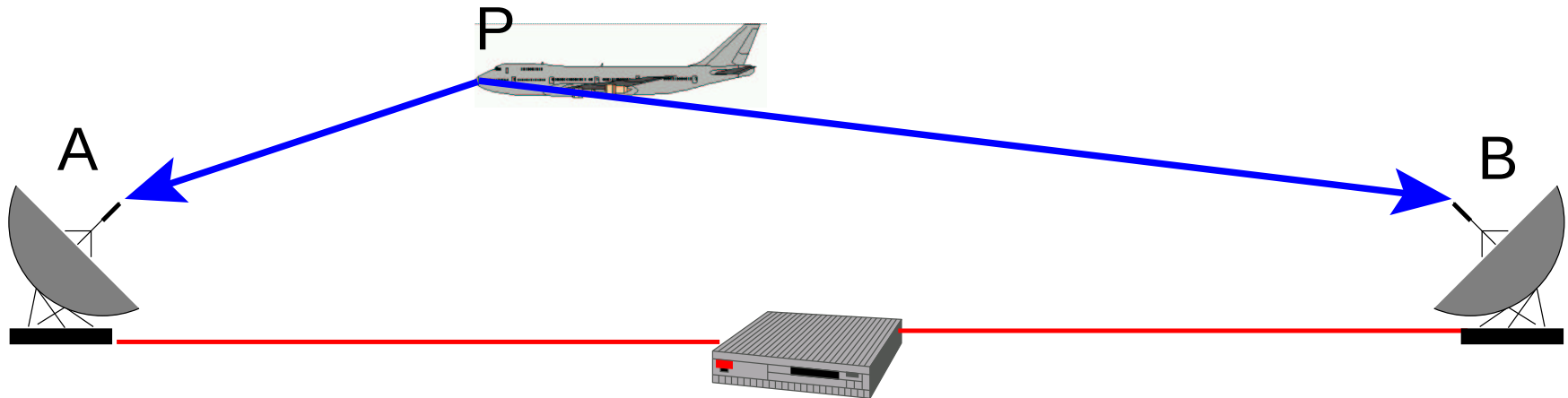
Engineering HOT SPOT

- Positioning with ultrasound, radar pings etc.
- WW2 radar. Henk Muller in computer science etc.



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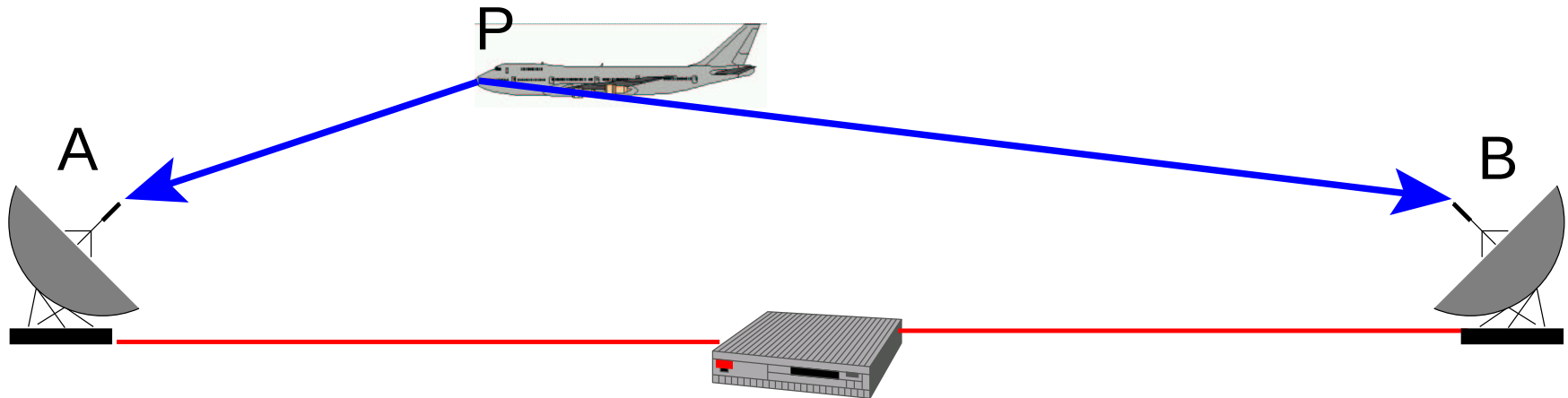


- Elapsed time between signals: $t = t_A - t_B$.

$$|\mathbf{a} - \mathbf{r}| - |\mathbf{b} - \mathbf{r}| = ct \quad (\text{wave speed } c)$$

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$$|\mathbf{a} - \mathbf{r}| - |\mathbf{b} - \mathbf{r}| = ct \quad (\text{wave speed } c)$$

- Vector equation of a hyperbola!

Homework

- Read *James* pages
 - 206–211
 - 217–237
 - 238–246
- Go through worked examples
 - 4.8–4.13
 - 4.19–4.44
- Do exercises
 - 4.2.6 nos. 1,2
 - 4.2.8 nos. 18,19
 - 4.2.10 nos. 32,33
 - 4.2.12 nos. 44,45
 - 4.6 review exercises 1,7,11,14,19